Your Signature ____________________ *This is a closed book exam. Calculators are permitted. There are five problems. Each problem is worth 10 points, for a total of 50 points.*

Show all your work. Correct answers with insufficient or incorrect work will not get any credit.

Score

1.	(20)	
2.	(20)	
3.	(20)	
4.	(20)	
5.	(30)	
Total.	(110)	

Sheets attached:____________________

1. A box contains 10 coins, of which 5 are fair and 5 are biased to land heads with probability 0.7. A coin is drawn from the box and tossed once.
   (a) What is the chance that it will land a head?
   (b) Suppose that the coin drawn landed a head. Given this information, what is the conditional probability that if I draw another coin from the box(without replacing the first coin), then that coin will be a fair coin ?

2. Let $X_1, \dots X_n, \dots$ be a sequence of independent and identically distributed random variables, with $E(X_i) = 10$ and $V(X_i) = 1$.
   (a) State the weak law of large numbers for the above mentioned sequence $X_1, \dots X_n, \dots$.
   (b) Let $T_n = \frac{10}{n} \sum_{i=1}^{n} (30 + X_i)$. Does T_n converge to something and if so what is type of convergence ?

3. Let X be a $\Gamma(4, 2)$ random variable with moment generating function $M_X(t)$. Let Y be another random variable with moment generating function $M_Y(t)$. Suppose $M_X(t) = M_Y(4t)$, then
   (a) Find the relationship between X and Y.
   (b) Find the probability density function of Y.

4. Waiting times at a service counter in a pharmacy are exponentially distributed with a mean of 10 minutes.
   (a) What is the probability that one customer has to wait for more than 10 minutes ?
   (b) Let 100 customers come to the service counter in a day. Let S_{100} be the number of customers that wait for more than 10 minutes.
      (i) Write out the probability mass function of S_{100}.
      (ii) Using the central limit theorem, approximate the probability that at least half of the customers that arrive in a day must wait for more than 10 minutes. Please explain clearly how the central limit theorem is used in the solution.

5. Let X and Y be independent random variables each geometrically distributed with parameter p.
   (a) Find $P(\min(X, Y) = X)$.
   (b) Find the distribution of $X + Y$.
   (c) Find $P(Y = y | X + Y = z)$.